Source of Allergies Examination

1. You wish, for some perverse reason, to buy some wire to make a circle of diameter 1 foot. What is the shortest length of wire should you buy?

   (A) 2 feet **(B)** 3 feet **(C)** 4 feet **(D)** 4.1 feet **(E)** That depends on the examiner.

2. An intern calculates the single benefit premium for a whole life insurance of 100,000 dollars on a person aged 40 years and 4 months payable at the end of the year of death. The intern calculates the premium using the Illustrated Life Table with $i = 0.06$ and the assumption of uniform distribution of deaths over each year of age. She comes up with a premium of A.

   Next day her boss has a conniption. He gives her a lecture on the importance of modeling and insists that she redo the calculation using hyperbolic interpolation. She obeys and comes up with a premium of B. Calculate $A - B$.

   (A) Less than 15 cents **(B)** Between 15 cents and a dollar **(C)** Between a dollar and a hundred dollars **(D)** Between a hundred dollars and a thousand dollars **(E)** Over a thousand dollars.

3. The force of mortality for ordinary mortals is a constant 0.02. Those who take the actuarial exams experience an additional force that starts at 0 when they start preparing for the exams and increases at a steady rate of 1 per year for the first year and decreases at a steady rate down to zero at the end of 2 years. Then they get used to the stress and mortality reverts to the standard rate. Calculate the ratio of the expectation of the curtate future lifetime of a 22 year old who is starting to prepare for the actuarial exams to that of an ordinary mortal who is not going to take the exams.

   (A) 0.4 **(B)** 0.5 **(C)** 0.6 **(D)** 0.7 **(E)** 0.8

4. For a semicontinuous 10-payment whole life insurance of 256,366 dollars on (31) the premium is set by the equivalence principle. You are given:

   - Mortality follows the De Moveover's law which says:

- The insured shall not die until all the premiums payments have been made.
- After that the insured is free to die at any time with equal probability until the age of 120, at which age everybody will be declared dead.

- Force of usury is 1.

Calculate the benefit premium.

(A) One dime **(B)** Two bits **(C)** One dollar **(D)** Ten dollars **(E)** One C note

5. There are three causes that can make a person quit taking exams:

   (1) A nervous breakdown from trying to avoid integration by parts

   (2) Getting a less stressful job like driving a taxi around Place de la Concorde

   (3) Other causes that will remain a mystery.

   You are freely given:

   $${}_tq_x'^{(1)} = {}_tq_{x+1}'^{(1)} = 0.5\,t,\ {}_tq_x'^{(2)} = {}_tq_{x+1}'^{(2)} = 0.1\,t,\ {}_tq_x'(3) = {}_tq_{x+1}'^{(3)} = 0.01\,t;\ 0 < t \le 1.$$

   Calculate the probability that (x) will quit during the second year because of getting a less stressful job.

   (A) 0.023 **(B)** 0.028 **(C)** 0.033 **(D)** 0.038 **(E)** 0.043

6. Lucky Tom lost his job as a walker for SOA/CAS and is panhandling in front of Union Station. People pass him by at a Poisson rate of 15 per minute. 60% of the people give him nothing, 20% give him some change but no more than a quarter and the rest give him more than a quarter. Calculate the probability that over a period of 20 seconds somebody will give Lucky Tom some money.

   (A) 0.1 **(B)** 0.3 **(C)** 0.5 **(D)** 0.7 **(E)** 0.9

7. Methuselah, aged 900, applies for whole life insurance to the amount of 100,000. The insurance company assigns his case to a student preparing

for the Joint Exam 3. Since no mortality data for people aged 900 are available, the student, who is very good at modeling - whose name is also Heidi - decides to go with constant force of mortality. Since people may or may not live to be 900, the student decides that the probability that one could survive 900 years will be set to 1/2. On this basis she calculates the force of mortality. Assuming that benefit will be paid at the moment of death and that the premiums will be paid continuously, Heidi calculates the benefit premium.

Unbeknownst to Heidi, Methuselah's destiny had changed and along with it the density. It turns out that he could die at any time over the next 70 years.

With this new information, using $\delta = 0.077$, calculate the expected value of the loss at issue.

(A) Between 10,000 and 15,000 **(B)** Between 15,000 and 20,000 **(C)** Between 20,000 and 25,000 **(D)** Between 25,000 and 30,000 **(E)** Over 30,000

8. Tyrannosaurus Max has an infinite capacity to store calories. After having feasted upon a large number of actuarial scientists and washed them down with a great amount of beer, Max retires to his bed in a stupor. Unable to sleep through the night because of discomfort, he wakes up periodically. The durations of sleep between successive fits of waking up are independent and exponentially distributed with mean 1 hour. The causes of waking up can be classified as Type 1 (an imagined shortcut to solve an exam question) and Type 2 (nausea) independently of each other. On the average Type 2 occurrences are half as frequent as Type 1 occurrences. Given that Max woke up exactly once between 10 P.M. and 10:30 P.M. calculate the probability that he woke between 10:15 and 10:24 and that the purpose was disgorgement.

   (A) 0.1 **(B)** 0.15 **(C)** 0.20 **(C)** 0.25 **(D)** 0.30 **(E)** 0.35

9. Jamie likes to stay in his room and listen to CDs by a rap musician. In this artiste's work, offensive words occur at a Poisson rate of 20 per minute of music. If his mother hears any of these words she will forbid him from playing that CD. Jamie always keeps the door to his room closed. When an offensive word occurs there is a 10% chance that

Jamie's mother will hear it. Jamie has just put on a CD. Calculate the probability that within 30 seconds his mother will scream to turn it off.

(A) 0.4 **(B)** 0.5 **(C)** 0.6 **(D)** 0.7 **(E)** 0.8

10. The above mentioned Jamie grew up and begat Amy. When Amy turned 3, her parents asked her what she wanted to study when she grew up. Amy said, "Mortuary science." Perhaps because the parents were hard of hearing from listening to loud music, they thought Amy said, "Actuary Science." And they were proud.

    They decided, starting then, to put away a sum of money at the beginning of each year for the next 15 years. After 15 years Amy will start taking the actuarial exams. Starting 15 years from now the fund will pay at the beginning of every year for 20 years so long as she is taking exams, a sum of 50,000 dollars to defray the cost of exam fees, books, study manuals and seminars.

    If at any time during the 15 years while the fund is being collected Amy decides not to take the exams, Amy will get all the money accumulated in the fund at the end of the year of her decision.

    The following information is from the Boring Life Table (BLT, developed by Professor Alan Boring, who based his calculation on the hypothesis that end of taking exams is end of life and $i = 0.06$.):

    - Nobody faces biological death
    - $\ddot{a}_{18:\overline{20|}} = 4.7$

    Oh, another thing; You are also further given:

    - The actuarial present value of the fund will equal the actuarial present value of payments to Amy

    Calculate the reserve at the end of 14 years.

    (A) 200,000 **(B)** 210,000 **(C)** 220,000 **(D)** 230,000 **(E)** 240,000

11. Phat Trimmer, as the Common Executive Officer of a large corporation, enjoys certain perks. Every Friday afternoon he may go at the company's expense to a massage parlor or a casino. If he is at a casino

this week, the probability that he will go to the casino again next week is one third of the probability that he will go to the massage parlor next week. If he is at a massage parlor this week, he is equally likely to be at the casino or the massage parlor next week. Every week the probability that he will be fired by Thursday is 0.2.

On each visit to the casino, Phat blows 5,000 dollars. Each visit to the massage parlor costs 1,000 dollars including tip.

The company will pay the expenses but only in multiples of hundreds.

As of the end of this Thursday, June 3, Phat has not been fired. On June 4, he is equally likely to be at the massage parlor as at the casino. Calculate the expected amount the company pays for his Friday amusement over the month of June.

(A) 5,600 **(B)** 5,700 **(C)** 5,800 **(C)** 5,900 **(D)** 6,000 **(E)** 6,100

Solutions

1. Of course you need π feet. If you want to make a circle of diameter 1 foot, 3 feet won't cut it. But the SOA will want you to round to the nearest integer. So even though the correct answer is C , if you wish to pass you have to say B.

   Answer: B

2. We need

   $$100,000\, A_{40\,\frac{1}{3}} = 100,000v^{2/3}\left(\,{}_{\frac{2}{3}}q_{40\,\frac{1}{3}} + {}_{\frac{2}{3}}p_{40\,\frac{1}{3}}\, A_{41}\right) = 100,000v^{2/3}\left\{1 - {}_{\frac{2}{3}}p_{40\,\frac{1}{3}}\,(1 - A_{41})\right\}.$$

   From the ILT, $A_{41} = 0.16869$ and $q_{40} = 2.78(10)^{-3}$.

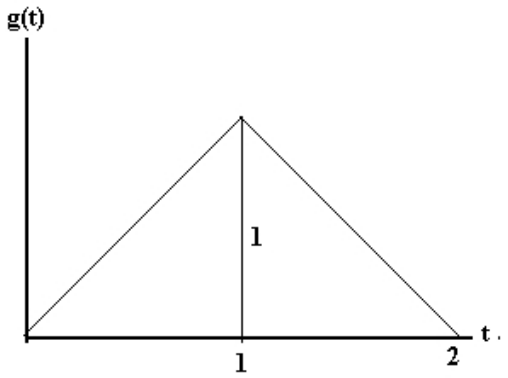
   $$A{-}B = 100,000(1.06)^{-2/3}\left\{\frac{(2/3)q_{40}}{1-(1/3)q_{40}} - (2/3)q_{40}\right\}(1{-}0.16869) = 0.1374.$$

   Answer: A

3. For the ordinary person,

   $$e_x = e^{-0.02} + e^{-0.04} + \cdots = \frac{e^{-0.02}}{1-e^{-0.02}} = 49.502.$$

   Let $g(t)$ be the additional force of mortality for the person who is

   preparing for the exam. Then the graph of $g(t)$ looks like the figure below. Clearly from the areas of the triangles.

   $$\int_0^1 g(t)\,dt = 0.5 \quad \text{and} \quad \int_0^2 g(t)dt = 1.$$

Hence

$$\begin{aligned} p_{[22]} &= e^{-0.02}e^{-0.5} = e^{-0.52} = 0.5945 \\ {}_2p_{[22]} &= e^{-0.04}e^{-1} = e^{-1.04} = 0.35345 \\ e_{[22]} &= p_{[22]} +_2 p_{[22]}(1 + e_{24}) \\ &= 0.5945 + 0.35345(1 + 49.502) = 18.44 \end{aligned}$$

The desired ratio is

$$18.44/49.502 = 0.37.$$

Answer: A

4. Suppose the time of death is T. The present value of the benefit is $256366\, e^{-T}$. The distribution of T is, $Pr(T \le 9) = 0$ and for $9 < t < 89$, $f(t) = 1/80$. Therefore the APV of the benefits is

$$256366(1/80)\int_9^{89} e^{-t} = 256366(1/80)(e^{-9} - e^{-89}) = 0.39548.$$

If P is the premium, then the present value of the premiums is

$$P\,\frac{1 - e^{-10}}{1 - e^{-1}} = 1.582P.$$

Equating the two, $P = 0.39548/1.582 = 0.25$.

Answer: B

5.

$$\begin{aligned} p_x^{(\tau)} &= (0.5)(0.9)(0.99) = 0.4455 \\ q_{x+1}^{(2)} &= (0.1)\int_0^1 (1 - 0.5t)(1 - 0.01t)\, dt = 0.0747 \end{aligned}$$

The desired probability is $(0.4455)(0.0747) = 0.03326$.

Answer: C

6.

$$f_S(0) = \exp\{-\lambda(1 - f_X(0))\} = e^{-2}.$$

The desired probability is $1 - e^{-2} = 0.86$.

Answer: E

7. First calculate μ from

$$e^{-900\mu} = 0.5$$

or $\mu = 7.7016(10)^{-4}$. Since the future lifetime has an exponential distribution, the benefit premium is

$$100,000\,\mu = 77.$$

With the revised mortality

$$\begin{aligned} L &= \left(100,000 + \frac{P}{\delta}\right) e^{-\delta T} - \frac{P}{\delta} \\ &= 101,000\, e^{-0.077T} - 1,000 \\ E(L) &= 101,000\,\frac{1}{70}\int_0^{70} e^{-0.077t}\,dt \;-\; 1,000 = 17,653. \end{aligned}$$

Answer: B

8. Given that the call arrived in an interval its time of arrival is uniformly distributed over that interval. So given that the call arrived in an interval of 30 minutes, the probability that call arrived in a sub-interval of 9 minutes is 9/30. And the probability that it was a Type 2 call is 1/3. Hence the desired probability is 0.1.

Answer: A

9. This is a Compound Poisson problem with $\lambda = 10$ and $Pr(X = 0) = 0.9$. So the probability that the mother will hear at least 1 is

$$1 - e^{-\lambda(1-f_X(0))} = 1 - e^{-1} = 0.63.$$

Answer: C

10. This is a 15-year deferred 20-year temporary annuity of 50,000 on (3). Let P be the yearly deposit in the fund. Then

$$P = \frac{50,000\, v^{15}\, \ddot{a}_{18:\overline{20|}}}{\ddot{a}_{\overline{15|}}}$$

The reserve at the end of 14 years is given by

$${}_{14}V = 50,000\, v\, (4.7) \;-\; P \;=\; 212173.$$

Answer: B

11. If the states are: (0) the massage parlor, (1) the casino and (2) joblessness, then the transition matrix is

$$\begin{pmatrix} 0.4 & 0.4 & 0.2 \\ 0.6 & 0.2 & 0.2 \\ 0 & 0 & 1 \end{pmatrix}.$$

The probability matrices are:

$$\begin{aligned} \mathbf{P}_0 &= (0.4 \;\; 0.4 \;\; 0.2);\ \mathbf{P}_1 = (0.304 \;\; 0.208;\ 0.488) \\ \mathbf{P}_2 &= (0.2464 \;\; 0.1632 \;\; 0.5904);\ \mathbf{P}_3 = (0.19648 \;\; 0.1312 \;\; -) \end{aligned}$$

The expected payment is

$$(0.4+0.304+0.2454+0.19648)(1000)+(0.4+0.208+0.1632+0.1312)(5000) = 5658.$$

My answer is A. But then, according SOA examiners' rule of rounding, the answer has to be B. Who am I to argue?